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Wigner distributions of quarks for different polarizations

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Abstract We calculate quark Wigner distributions using the light-front wave functions in a dressed quark model. In this model, a proton target is replaced by a simplified spin-1/2 state, namely a quark dressed with a gluon. We calculate the Wigner distributions for different polarization configuration of quark and the target state in this model.

Keywords Wigner distributions · dressed quark model

1 Introduction

Generalized parton distributions (GPDs) and transverse momentum dependent parton distributions (TMDs) [1] give the three-dimensional picture of the hadronic structure. GPDs [2; 3] essentially can provide an understanding of the orbital angular momentum of partons inside the hadrons. GPDs and TMDs are encompassed in a correlation function which describes off-forward scattering amplitudes that were introduced in Ref. [4]. For certain kinematics, if one calculates the matrix element at $z^+ = 0$ or $z^- = 0$, one obtains a reduced matrix element called as generalized transverse momentum dependent parton distributions (GTMDs) [1; 4; 5]. Wigner distributions are obtained by taking Fourier transform of GTMDs which are also called as mother distribution functions. The main goal is to understand the partonic structure of nucleon that can be achieved by means of position-momentum distributions like Wigner distributions. In a particular limit, these distributions can be related to the spin and angular momentum of quarks and gluons. Wigner distribution is defined as matrix elements between different nucleon states as a function of three position and three momentum of a quark or a gluon inside the nucleon in a particular polarization state.

Quark and gluon Wigner distribution as a function of three position and three momentum variables was first studied by authors in Refs. [6; 7] in which relativistic effects were neglected. Wigner distributions [8] do not give probabilistic interpretation as they are not positive definite. Only in the classical limit, they become positive definite. In spite of this, a phase-space distribution are worth exploring as they can be related to the measurable quantities by integration over transverse position or momentum of the parton. The model dependent calculation of Wigner distributions may provide us information

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about the correlators of quarks and gluons in a nucleon. Especially, quark orbital angular momentum can be accessed from the Wigner distribution for unpolarized quarks in the longitudinally polarized nucleon (F_{14}) [9; 10; 11; 12; 13; 14]. These calculations are also a boon in estimating relations of TMDs and GPDs using GTMDs in a certain limit. Wigner distributions have been studied in various phenomenological models [10; 14; 15; 16; 17; 18; 19; 20].

In this work, we investigate the quark Wigner distributions in light-front Hamiltonian formalism [21]. In this formalism, one gets an intuitive picture of deep inelastic scattering for non-collinear, interacting and massive partons. The target state is a dressed quark state and is expanded in terms of light-front wavefunctions (LFWFs) which are the multiparton wavefunctions. So, in dressed quark model we replace the proton state as a quark dressed with a gluon which incorporates gluonic degrees of freedom in this perturbative model.

The plan of this paper is as follows. In Sec. 2, we calculate the Wigner distribution of quark. For different polarization configuration of quark and target state, one can define 16 Wigner distributions at the leading twist. All these Wigner distributions are calculated in light cone dressed quark model using Hamiltonian perturbation theory. In our model, we obtained 8 independent distributions and we illustrate here only 4 of them, a complete discussion of all the leading twist Wigner distributions can be found in [23]. In Sec. 3 we give numerical results and three-dimensional plots of the independent distributions in momentum space, impact parameter space and mixed space. Finally, we summarize our results in Sec. 4.

2 Quark Wigner Distributions in Dressed Quark Model

The Wigner distribution of quarks is defined as [1; 9]

$$\rho^{[I]}(b_\perp, k_\perp, x, s, s') = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot b_\perp} W^{[I]}(\Delta_\perp, k_\perp, x, s, s') \quad (1)$$

where, b_\perp is the impact parameter space conjugate to momentum transfer (Δ_\perp) of a dressed quark in the transverse direction. GTMDs are defined through quark-quark correlator $W^{[I]}$ at a fixed light-front time as

$$W^{[I]}(\Delta_\perp, k_\perp, x, s, s') = \int \frac{dz^- d^2 z_\perp}{2(2\pi)^3} e^{ik \cdot z} \left\langle p^+, \frac{\Delta_\perp}{2}, s \left| \bar{\psi}\left(-\frac{z}{2}\right) \Omega \Gamma \psi\left(\frac{z}{2}\right) \right| p^+, -\frac{\Delta_\perp}{2}, s' \right\rangle \Big|_{z^+=0} \quad (2)$$

The initial and final dressed quark states are defined in the symmetric frame, with the longitudinal momentum as p^+ , the transverse momentum transfer is Δ_\perp and s, s' are the helicities of initial and final target state. The average four momentum of the quark is k , with $k^+ = xp^+$, here x is the longitudinal momentum fraction of the parton. Ω is the gauge link which is required for color gauge invariance and is chosen to be unity.

The state of a dressed quark with momentum p and fixed helicity s can be written in terms of multiparton wavefunctions (LFWFs) as the expansion of the Fock state [21]

$$\begin{aligned} |p^+, p_\perp, s\rangle &= \Phi^s(p) b_s^\dagger(p) |0\rangle + \sum_{s_1 s_2} \int \frac{dp_1^+ d^2 p_1^\perp}{\sqrt{16\pi^3 p_1^+}} \int \frac{dp_2^+ d^2 p_2^\perp}{\sqrt{16\pi^3 p_2^+}} \sqrt{16\pi^3 p^+} \delta^3(p - p_1 - p_2) \\ &\quad \times \Phi_{s_1 s_2}^s(p; p_1, p_2) b_{s_1}^\dagger(p_1) a_{s_2}^\dagger(p_2) |0\rangle \end{aligned} \quad (3)$$

$\Phi^s(p)$ is the single quark state and $\Phi_{s_1 s_2}^s(p; p_1, p_2)$ is the quark-gluon LFWF. s_1 and s_2 are helicities of quark and gluon respectively. $\Phi^s(p)$ gives the wavefunction normalization of the quark. $\Phi_{s_1 s_2}^s(p; p_1, p_2)$ gives the probability amplitude to find a bare quark (gluon) with momentum $p_1(p_2)$ and helicity $s_1(s_2)$ inside the dressed quark. Using the Jacobi momenta

$$k_i^+ = x_i P^+ \text{ and } k_i^\perp = q_i^\perp + x_i P^\perp \quad \text{so that} \quad \sum_i x_i = 1, \quad \sum_i q_{i\perp} = 0 \quad (4)$$

the two particle LFWF can be written in terms of boost invariant LFWF as

$$\sqrt{P^+} \Phi(p; p_1, p_2) = \Psi(x_i, q_i^\perp) \quad (5)$$

Using the two particle LFWF [21] and two component formalism [22] at leading twist, one obtains only four Dirac operators $\Gamma = \{\gamma^+, \gamma^+ \gamma^5, i\sigma^{+1} \gamma^5, i\sigma^{+2} \gamma^5\}$ which corresponds to Wigner distributions for unpolarized, longitudinally polarized and transversely polarized dressed quark. So the quark-quark correlator using two particle LFWFs for different polarization at twist-2 are given by

$$W^{[\gamma^+]}(\Delta_\perp, k_\perp, x, s, s') = \sum_{\lambda'_1, \lambda_1, \lambda_2} \Psi_{\lambda'_1 \lambda_2}^{*s'}(x, q'^\perp) \chi_{\lambda'_1}^\dagger \chi_{\lambda_1} \Psi_{\lambda_1 \lambda_2}^s(x, q^\perp) \quad (6)$$

$$W^{[\gamma^+ \gamma^5]}(\Delta_\perp, k_\perp, x, s, s') = \sum_{\lambda'_1, \lambda_1, \lambda_2} \Psi_{\lambda'_1 \lambda_2}^{*s'}(x, q'^\perp) \chi_{\lambda'_1}^\dagger \sigma_3 \chi_{\lambda_1} \Psi_{\lambda_1 \lambda_2}^s(x, q^\perp) \quad (7)$$

$$W^{[i\sigma^{+j} \gamma^5]}(\Delta_\perp, k_\perp, x, s, s') = \sum_{\lambda'_1, \lambda_1, \lambda_2} \Psi_{\lambda'_1 \lambda_2}^{*s'}(x, q'^\perp) \chi_{\lambda'_1}^\dagger \sigma_j \chi_{\lambda_1} \Psi_{\lambda_1 \lambda_2}^s(x, q^\perp) \quad (8)$$

with σ_i are the three Pauli matrices. Eqs. (6), (7) and (8) gives unpolarized, longitudinally polarized and transversely polarized GTMDs in terms of LFWFs. For various combinations of unpolarized (U), longitudinally polarized (L) and transversely polarized (T) target and quark states, the quark-quark correlators can be parametrized into 16 Wigner distributions [18] at leading twist. We denote Wigner distributions by $\rho_{\lambda, \lambda'}$, where λ and λ' represents polarization of the target state and quark respectively. In this model, we have 10 independent Wigner distributions out of which $\rho_{TT}^j(\mathbf{b}_\perp, \mathbf{k}_\perp, x)$ is zero with $j = 1, 2$ and $\rho_{LU} = \rho_{UL}$. However, we discuss only four independent Wigner distributions but the details of all the 10 Wigner distributions can be looked in Ref [23] and in that we have classified Wigner distribution in terms of polarization of the target states.

The unpolarized Wigner distribution

$$\rho_{UU}(\mathbf{b}_\perp, \mathbf{k}_\perp, x) = \frac{1}{2} \left[\rho^{[\gamma^+]}(\mathbf{b}_\perp, \mathbf{k}_\perp, x, \hat{e}_z) + \rho^{[\gamma^+]}(\mathbf{b}_\perp, \mathbf{k}_\perp, x, -\hat{e}_z) \right] \quad (9)$$

The unpolarized-longitudinally polarized Wigner distribution

$$\rho_{UL}(\mathbf{b}_\perp, \mathbf{k}_\perp, x) = \frac{1}{2} \left[\rho^{[\gamma^+ \gamma^5]}(\mathbf{b}_\perp, \mathbf{k}_\perp, x, \hat{e}_z) + \rho^{[\gamma^+ \gamma^5]}(\mathbf{b}_\perp, \mathbf{k}_\perp, x, -\hat{e}_z) \right] \quad (10)$$

The longitudinal Wigner distribution

$$\rho_{LL}(\mathbf{b}_\perp, \mathbf{k}_\perp, x) = \frac{1}{2} \left[\rho^{[\gamma^+ \gamma^5]}(\mathbf{b}_\perp, \mathbf{k}_\perp, x, \hat{e}_z) - \rho^{[\gamma^+ \gamma^5]}(\mathbf{b}_\perp, \mathbf{k}_\perp, x, -\hat{e}_z) \right] \quad (11)$$

The transverse Wigner distribution

$$\rho_{TT}(\mathbf{b}_\perp, \mathbf{k}_\perp, x) = \frac{1}{2} \delta_{ij} \left[\rho^{[i\sigma^{+j} \gamma^5]}(\mathbf{b}_\perp, \mathbf{k}_\perp, x, \hat{e}_i) - \rho^{[i\sigma^{+j} \gamma^5]}(\mathbf{b}_\perp, \mathbf{k}_\perp, x, -\hat{e}_i) \right] \quad (12)$$

The pretzelous Wigner distribution

$$\rho_{TT}^\perp(\mathbf{b}_\perp, \mathbf{k}_\perp, x) = \frac{1}{2} \epsilon_{ij} \left[\rho^{[i\sigma^{+j} \gamma^5]}(\mathbf{b}_\perp, \mathbf{k}_\perp, x, \hat{e}_i) - \rho^{[i\sigma^{+j} \gamma^5]}(\mathbf{b}_\perp, \mathbf{k}_\perp, x, -\hat{e}_i) \right] \quad (13)$$

Thus, the analytical expressions for the four Wigner distributions that we study are given by

$$\rho_{UU}(\mathbf{b}_\perp, \mathbf{k}_\perp, x) = N \int d^2 \Delta_\perp \frac{\cos(\Delta_\perp b_\perp)}{D(q_\perp) D(q'_\perp)} \left[\frac{(4k_\perp^2 - \Delta_\perp^2 (1-x)^2)(1+x^2)}{x^2(1-x)^3} + \frac{4m^2(1-x)}{x^2} \right] \quad (14)$$

$$\rho_{UL}(\mathbf{b}_\perp, \mathbf{k}_\perp, x) = N \int d^2 \Delta_\perp \frac{\sin(\Delta_\perp b_\perp)}{D(q_\perp) D(q'_\perp)} \left[\frac{4(k_x \Delta_y - k_y \Delta_x)(1+x)}{x^2(1-x)} \right] \quad (15)$$

$$\rho_{LL}(\mathbf{b}_\perp, \mathbf{k}_\perp, x) = N \int d^2 \Delta_\perp \frac{\cos(\Delta_\perp b_\perp)}{D(q_\perp) D(q'_\perp)} \left[\frac{(4k_\perp^2 - \Delta_\perp^2 (1-x)^2)(1+x^2)}{x^2(1-x)^3} - \frac{4m^2(1-x)}{x^2} \right] \quad (16)$$

$$\rho_{TT}(\mathbf{b}_\perp, \mathbf{k}_\perp, x) = N \int d^2 \Delta_\perp \frac{\cos(\Delta_\perp b_\perp)}{D(q_\perp) D(q'_\perp)} \left[\frac{2(4k_\perp^2 - \Delta_\perp^2 (1-x)^2)}{x(1-x)^3} \right] \quad (17)$$

where,

$$N = \frac{g^2 C_F}{2(2\pi)^2}, \quad C_F \text{ is the color factor}$$

$$D(q_\perp) = \left[m^2 - \frac{m^2 + (k_\perp + \frac{\Delta_\perp(1-x)}{2})^2}{x} - \frac{(k_\perp + \frac{\Delta_\perp(1-x)}{2})^2}{1-x} \right]$$

$$D(q'_\perp) = \left[m^2 - \frac{m^2 + (k_\perp - \frac{\Delta_\perp(1-x)}{2})^2}{x} - \frac{(k_\perp - \frac{\Delta_\perp(1-x)}{2})^2}{1-x} \right] \quad (18)$$

We observed that, $\rho_{TT}^\perp(\mathbf{b}_\perp, \mathbf{k}_\perp, x) = 0$ in this model.

3 Numericals and plots of probability densities in three dimension

As mentioned in the previous section, we study 4 independent Wigner distributions given in Eqs.(14)–(17) which are five dimensional function of b_x , b_y , k_x , k_y and x . We integrate over the momentum fraction x which can take values only between $(0, 1)$, in the dressed quark model. We observe the behavior in the transverse momentum and impact parameter space of these 4 distributions. One may also look at the mixed distributions by integrating out transverse momentum and transverse position along the perpendicular transverse directions. In the mixed distribution one expects distribution to be real as the remaining variables are not restricted by Heisenberg's uncertainty principle. In this section, we show the numerical results and the plots of the quark Wigner distributions in transverse momentum space with $b_\perp = 0.4 \text{ GeV}^{-1}$, transverse position space with $k_\perp = 0.4 \text{ GeV}$ and mixed space. The integration limit of Δ_\perp should ideally be from zero to infinity. However, in the numerical integration,

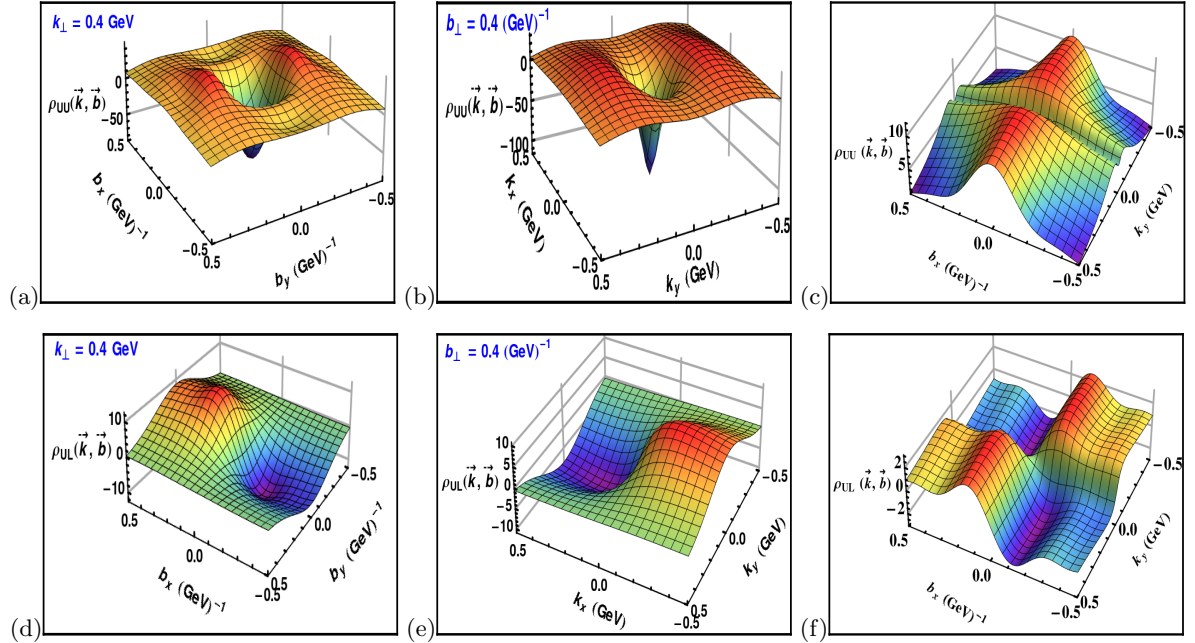


Fig. 1 3D plot of Wigner distributions ρ_{UU} and ρ_{UL} in \mathbf{k}_\perp -space, \mathbf{b}_\perp -space and $b_x - k_y$ space with $\Delta_{\perp max} = 20 \text{ GeV}$

we have used a cutoff on Δ_\perp integration, $\Delta_{\perp max} = 20 \text{ GeV}$. It is to be noted that in the previous work [14; 15], a lower value of $\Delta_{\perp max}$ was used and the results depend on this cutoff. We use Levin's method to perform numerical integration over x and Δ_\perp . It is a very effective method as it gives good convergence and the results do not depend on $\Delta_{\perp max}$.

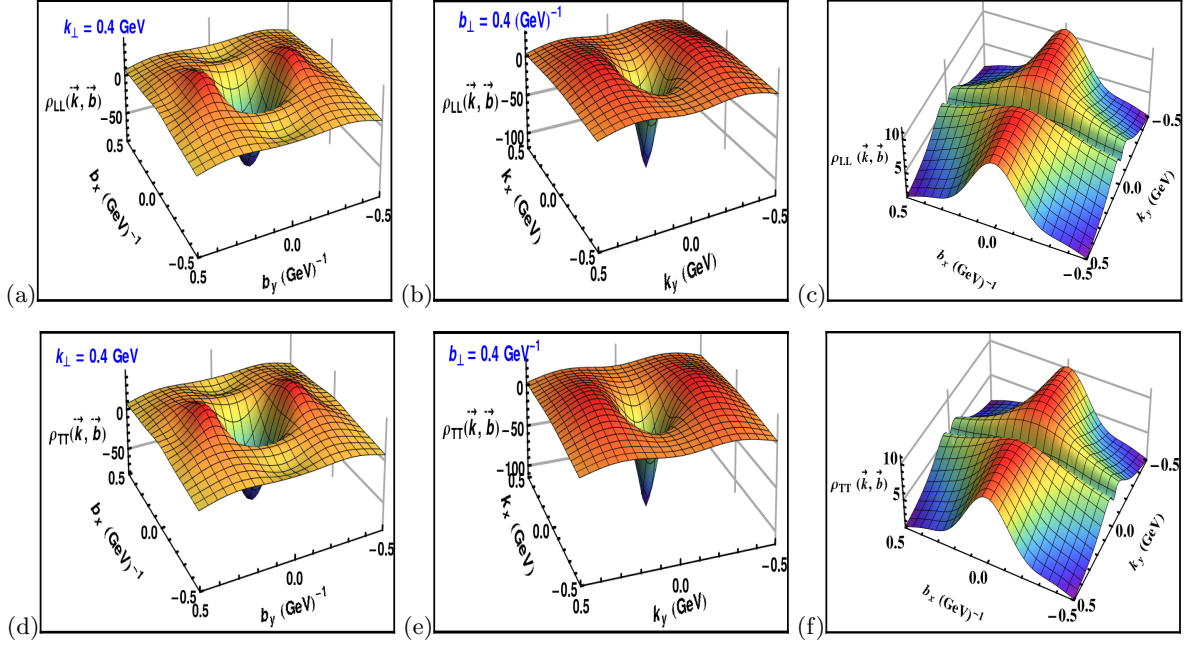


Fig. 2 3D plot of Wigner distribution ρ_{LL} and ρ_{TT} in \mathbf{k}_\perp -space, \mathbf{b}_\perp -space and $b_x - k_y$ space with $\Delta_{\perp max} = 20$ GeV

For numerical evaluation, in all the plots we have taken $m = 0.33$ GeV and $\Delta_{\perp max} = 20$ GeV. Fig 1(a) show a three-dimensional plot of Wigner distribution ρ_{UU} in \mathbf{k}_\perp -space with $\mathbf{b}_\perp = 0.4 \text{ GeV}^{-1} \hat{e}_y$. ρ_{UU} has a minimum and pick up negative value at $k_x = 0$ and $k_y = 0$. Fig 1(b) show the Wigner distribution ρ_{UU} in \mathbf{b}_\perp -space with $\mathbf{k}_\perp = 0.4 \text{ GeV} \hat{e}_y$. The behavior of \mathbf{k}_\perp -space is similar to \mathbf{b}_\perp -space with a different minima at $b_x = 0$ and $b_y = 0$ but it is still negative. Fig 1(c) show the Wigner distribution ρ_{UU} in mixed space where we have integrated out k_x and b_y dependence, thus we get the probability densities in the $b_x - k_y$ plane.

Fig. 1(d) shows a three-dimensional plot of Wigner distributions ρ_{UL} in momentum space with $\mathbf{b}_\perp = 0.4 \text{ GeV}^{-1} \hat{e}_y$. Fig. 1(e) shows a three-dimensional plot of Wigner distributions ρ_{UL} in impact parameter space with $\mathbf{k}_\perp = 0.4 \text{ GeV} \hat{e}_y$. Figs. 1(d) and (e) have similar nature. These two graphs show dipole structure as observed in Ref [9; 14]. Fig. 1(f) shows a three-dimensional plot of Wigner distributions in mixed space which shows the quadruple structure.

Fig 2(a)-(c) shows the three-dimensional plot of Wigner distributions ρ_{LL} in \mathbf{k}_\perp -space, \mathbf{b}_\perp -space and the mixed space respectively. Fig. 2 for ρ_{LL} shows similar nature as Fig 1 for ρ_{UU} as expected.

Fig. 2(d)-(f) shows the three-dimensional plot of Wigner distribution ρ_{TT} in \mathbf{k}_\perp -space, \mathbf{b}_\perp -space and mixed space respectively. In this case, both the quark and the target state are transversely polarized in the direction say x -direction. It is important to note that nature of ρ_{TT} is similar to ρ_{UU} and ρ_{LL} .

4 Conclusion

In this work, we include the transverse polarization of the quark and the target state unlike in the previous work [14]. In this model, we study only 4 independent quark Wigner distributions as illustration and the details will be given in Ref [23]. The unpolarized ρ_{UU} , longitudinally polarized ρ_{LL} and transversity distributions ρ_{TT} show similar nature. ρ_{LU} is equal to ρ_{UL} , which is related to orbital angular momentum of the quark and the pretzelous Wigner distribution ρ_{TT}^\perp is zero. We have used an improved method of numerical integration that gives better convergence and the results are independent of $\Delta_{\perp max}$.

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